Organization of networks with tagged nodes and biased links: *a priori* distinct communities. The case of Intelligent Design Proponents and Darwinian Evolution Defenders

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Abstract

Among topics of opinion formation it is of interest to observe the characteristics of networks with *a priori* distinct communities. As an illustration, we report on the citation network(s) unfolded in the recent decades through web available works belonging to selected members of the Neocreationist and Intelligent Design Proponents (IDP) and the Darwinian Evolution Defenders (DED) communities. An adjacency matrix of *tagged nodes* is first constructed; it is not symmetric. A generalization of considerations pertaining to the case of networks with *biased links*, directed or undirected, is thus presented.

The main characteristic coefficients describing the structure of such partially directed networks with tagged nodes are outlined. The structural features are discussed searching for statistical aspects, equivalence or not of subnetworks through the degree distributions, each network assortativity, the global and local clustering coefficients and the Average Overlap Indices. The various closed and open triangles made from nodes, moreover distinguishing the community, are especially listed to calculate the clustering characteristics. The distribution of elements in the *rectangular* submatrices are specially examined since they represent intercommunity connexions. The emphasis being on distinguishing the number of vertices belonging to a given community. Using such informations one can distinguish between opinion leaders, followers and main rivals and briefly interpret their relationships through psychological-like conditions intrinsic to behavior rules in either community. Considerations on other controversy cases with similar social constraints are outlined, as well as suggestions on further, more general, work deduced from our observations on such networks.

1 Introduction

In modern statistical physics nowadays [1], networks [2] underlying opinion formation of agents located at nodes [3], with links defined from data pertaining to econophysics [4] and/or sociophysics [5] have gathered much interest. There are many applications in line [6]. This has led to a flurry of theoretical works, most of them assuming a very large population of interacting agents. All these studies concern the organizational processes of populations, on networks or lattices [7,8,9]. However, not all agent based systems need to be studied on large scale-free networks, as if looking for some thermodynamic limit. On the contrary, finite size networks with agent small connectivity values are known to be more realistic [10,11].

Networks are usually composed of a large number of internal components, i.e. nodes and links, which can be used to describe a wide variety of systems of high intellectual and technological importance. Relevant questions pertain to the dynamics of collective properties, not only of agents on the network [12], but also for the network structure itself [13,14]. Recent results on the dynamics of social networks [15] suggest the occurrence of either discontinuous, continuous, and high order phase transitions and coexistence phase states in a large class of models [16]. This is also similar to features found in percolation and nucleation-growth problems.

Characteristics of small world networks (SWN) [17], as introduced by Watts and Strogatz might be close to describing social reality. It has been argued, though with more self-conviction than proof, that SWN are useful to describe population opinion switches [18]. Indeed opinion formation through *small* communities can be discussed on small networks [3] and still retain much value. Cases of interest pertain to the adherence, e.g., to religious denominations (or sects) [19], ideology struggles [20], language spreading [21] or policy analysis [22], as far studied off-network, like in a mean field approximation.

Up to now one has mainly searched for communities on networks inside rather restrictedly defined characteristics of node and/or link sets. However nodes could have several *a priori* tags. A generalization of the concept of networks to the case for example in which nodes are Ising-like spins has been already considered [18], - but rather from a strictly magnetic point of view, without obvious connexion to any societal case. Yet it is obvious that networks made of nodes with tags are very numerous, considering the status or the opinion of an individual, more generally speaking *agent*. Many examples of agents having even more than a binary set of *degrees of freedom* exist in on a network.

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The intensity of each tag can be also much varied, as in wealth, religious or language tags. Moreover links between tagged nodes of different natures might also occur.

Whence it is easily admitted that networks with *a priori* distinct communities are frequent [23,24,25]. For example, a society can be roughly considered to be composed of males and females. Other cases exist. All our readers belong to networks made of friends and enemies. A network can also be made of persons speaking one orseveral languages: restricting the present examples to mainly bilingual populations, or countries; see cases like Belgium, Ireland or Canada, - all having even marked differences. Networks of citizens belonging to one or another party ideology, like republicans and democrats, or leftists and rightists, are also common. A case of members with drastically different opinion is that of the Neocreationist and Intelligent Design Proponents (IDP) and the Darwinian Evolution Defenders (DED).

It is of general interest to observe the evolution of topical subjects pertaining to the nature of how science is perceived or understood, either by scientists or by the public at large [26], and why/how such opinions prevail and others disappear. One such topics is *creationism* indeed. It is opposed to aspects like Darwinian Evolution (DE). In recent years, creationism has been rejuvenated into a concept called Intelligent Design (ID), such that it pretends to be some kind of scientific alternative to DE and give some sort of interpretation to the big bang, and its consequences. This *per se* raises surely scientific (and other) questions [27]. The historical perspective is reduced here below to the main aspects pertinent to our present considerations: for comprehensive completeness those are only presented in Appendix A following [28].

No need to get further involved here in the pro and con, though one could, - being motivated by the affair, both as scientists and members of a monotheist culture. Several fundamental reasons why the subject is controversial, to say the least, have been often tackled in a mediatic way, hardly through unbiased scientific research. Letters (to the editors), papers, media appearance [29,30] by true scientists or politicians [31] or others are numerous on the subject for the last 15 years or so, enough to provide data to be analyzed within modern lines, as in statistical mechanics and scientometrics. Indeed much work in statistical physics attempts nowadays to reconcile intuitive or qualitative features, sometimes stylized facts, with simple models still driving into complexity aspects.

This DE-ID controversy subject of intense interest is thereby considered here below, from the point of view of two small world networks [17,18], making a larger one. Basically there are two communities, loosely connected, but with members hardly evolving in opinion but strongly arguing against the opposite one held by the other rival community. The following study is carried out by

analyzing the structural properties of the citation network unfolded in the recent decades (1990-2007) by web available works belonging to members of IDP and DED groups.

The data set acquisition and its limits are recalled in Sec. 2. We discuss 2 communities (or 2 opinions) existing on 2 subnetworks with approximately the same sizes (37 for IDP and 40 for DED). The methodology is based on constructing an adjacency matrix and examining the distribution of (i, j) elements in submatrices. We use notations like "directed links" (DL), i.e. those which obvioulsly have a direction from j to i and "undirected links" (UL) when *both* (i, j) and (j, i) linkages exist. Since the links may be, we emphasize, directional, but we do not consider their intensity nor frequency (thus there is no weight) nor timing of the citations (thus we do not consider time lags), we refer to these networks as made of merely *biased* links.

It seems rather appropriate to publish the whole adjacency matrix, following the data gathering methodology presented in [28]; see Appendix B. It might be expected to become as useful as the karate club data [32] (which had 44 nodes) or the the acquaintance network of Mormons (with 43 nodes) [33]. The network construction follows in lines with studies on large-scale networks, like co-authorship networks [34,35].

The structural aspects are next discussed searching for statistical aspects, equivalence or not of subnetworks, in Sec. 3. First it is searched whether it can be unambiguously proved that the degree distribution(s) or the link distribution(s) follow simple theoretical laws.

As a matter of fact, understanding some of the characteristics of the community made of the ID proponents and DE defenders requires accounting for their mutual interaction. As far as we know this aspect of the so called controversy has remained largely unexplored, except in [28], but appears of general scientific value, whatever the pro and con arguments on the intellectual ideas in the communities. Thus the present work aims to contribute to introducing a quantitative approach to the analysis of the interaction between two biased groups, in small networks. Let it be here pointed out that triplets of nodes are particularly examined in order to emphasize the inter- and intra-community connexions through one assortativity coefficient [36,37], two clustering coefficients [17] and one overlap index [38]. In order to do so, one has to generalize the nomenclature of triplets, whence of triangles, formed by nodes which can belong to two distinct communities and tied by UL or DL. The notation nomenclature convention aspect is shown in Fig. 8.

Practically for such a study, i.e. when considering networks with distinct communities, it is necessary to generalize the usual characteristic coefficients of networks describing either their structure or some dynamics to the present

case of so called networks with tagged nodes and biased links. Using such informations one can distinguish between leaders, rivals and followers. Moreover having in mind some *presumably accepted behavioral rules* of both communities, one can briefly interpret the relationships through the usual ethics and adequate psychological conditions of the members of either group.

In Sec. 4, some further discussion on the statistical mechanics of this illustrative case is presented in line with general considerations. Since it is intuitively obvious that there is no phase transition to be expected, in the sense of [3] because the members of such communities are pretty much behaving as in a very deep potential well, one has not to elaborate here on the dynamics of opinion formation. There is neither much change nor fluctuations in the node state, whatever the link weights or number. However it can be imagined that the opinions can evolve if considerations as outlined in [39] are taken into practice. They are reformulated for the present context in the conclusion section.

No need to say that most of the ideas so below developed can encompass many (rigid) networks, not only the DE-ID controversy, but also others formed by scientists with *a priori* constraints on rival scientific work. Moreover the present study suggests to examine more general cases in which the nodes have many tags, with different intensity ranges, whence systems which belong to universal classes other than the symmetric 1/2-Ising spin.

2 The data set

First the main actors of the ID movement were selected through the ID web pages and their corresponding links so far helped by an article by Pennock [40]. Next, a search of citations was made through the Scholar Google search engine. Directed and undirected interactions between agents are found through citations of the other (or one's own) ideas, as expressed through various media. We selected 37 IDP and 40 DED [28]. The examined time range goes from Oct. 01 till Nov 15, 2007.

In so doing, an adjacency matrix $M = (m_{ij}) \in R^{77\times 77}$ is built, see Appendix B. The matrix elements m_{ij} take the value 1 or 0 depending on whether or not a citation of i by j has taken place. Therefore, the number of links going into a node i, i.e. the $in-degree$, represents the number of authors that cite i. The number of links exiting from a node j, i.e. the $out-degree$, represents the number of authors that j cites. First we accepted self-citations, but they can be disregarded, see below. Recall that we consider "directed links" (DL) and "undirected links" (UL). In brief, the existence of a DL from j to i - beyond meaning that j cites i, implies that j is *never* cited by i. The number of citations might be asymmetric, but here no weight is given to a link nor

to a *direction*: the number of citations and their timing are here taken as irrelevant. The resulting graphs are thus of the binary directed nonweighted network types. The interconnectivity between different communities presented as encounter frequency, describing the probability a person encountering a stranger in another community has been discussed in [25]. However in the present case the encounter is very biased. Thus a discussion and variant of [25] is left for further work.

Obviously the matrix M is square but not symmetric. For ease of the data analysis, we have gathered in the low ranks of M the IDP (rows and columns from 1 till 37), and the DED in the upper ranks (rows and columns from 38 till 77). Each agent has received an arbitrary index in its subset. By writing

$$M = \begin{pmatrix} C & A \\ B & D \end{pmatrix} \tag{1}$$

we evidence the two possible communities or subgraphs described respectively by matrices C (= *creationists*) and D (= *Darwinians*), i.e. $C \in R^{37\times37}$ contains all the intra-citations among the IDP community; $D \in R^{40\times40}$ contains the intra-citations among the DED community, but A and B are rectangular matrices describing intercommunity links.

For further discussion, let us also introduce the matrix M_0 such that all diagonal terms are 0, i.e. not considering any self-citation, i.e. we define

$$M_0 = \begin{pmatrix} C_0 & A \\ B & D_0 \end{pmatrix} \tag{2}$$

It is also of interest to define a matrix emphasizing the links between communities, i.e. the matrix

$$F = \begin{pmatrix} 0 & A \\ B & 0 \end{pmatrix}. \tag{3}$$

Figs. 1-4 display the corresponding C_0 and D_0, A, B and M_0, networks respectively. Arrows indicate DL. They point from j to i, on an (i, j) link. There are 307 links, i.e. 102 among IDP, 86 among DED. Among these we notice that there are 26 self-citations, leading to so called self-loops, i.e. 11 in IDP, 15 in DED, i.e. amounting to a superfluous 11% and 17% contribution respectively, - they will be neglected here below. It remains a total of 281 links, i.e. 91 in the IDP and 71 in the DED communities respectively, thus for a total of 162

matrix	number of links	trace	DL	UL
M	307	26	219	31
C	102	11	79	6
D	86	15	51	10
C_0	91	0	79	6
D_0	71	0	51	10
A	86	0	86	0
B	33	0	33	0
F	119	0	89	15
M_0	281	0	219	31

Table 1
Global description of the relevant matrices: trace, i.e. number of self-citations, number of directed (DL) or undirected (UL) links. The total number of links is always for each matrix equal to the number of directed links, plus the double of the number of undirected links, plus the trace (self-citations), i.e. the number of finite elements in the matrix

(thus $\sim$ 57.7%), and 119 (thus $\sim$ 42.3%) inter-community links, indicating at once the relative importance of the inter-community exchanges. Among these 89 are DL (thus $\sim$ 74.8%) and 15 are UL (thus $\sim$ 12.6%) in other words members from the other opinion community do not tend to go somewhat unnoticed, but most arguing exchanges are rather concentrated among a few ($\simeq$ 15) opinion members. For completeness, let it be observed that there are 219 DL, but only 31 UL for the whole data set: 6 UL among IDP, 10 UL among DED, - an indication of internal community support, and a rough estimate of the number of opinion leaders; the other 15 UL concern cross-citing among the two groups as mentioned here above. For further reference we report the detailed data in Table 1.

3 Data statistical analysis

After building the IDP and DED networks and the overall network of agents, due to links through the empirical observation outlined here above, we proceed performing some classical structural analysis on such networks, i.e. an analysis of the node degree k_i distribution, but taking into account the directionality of the links through the out-degree k_i^{out} and in-degree k_i^{in} distributions, plus indicative coefficients, i.e. the network assortativity, the (global and local) clustering coefficients and the Average Overlap Index.

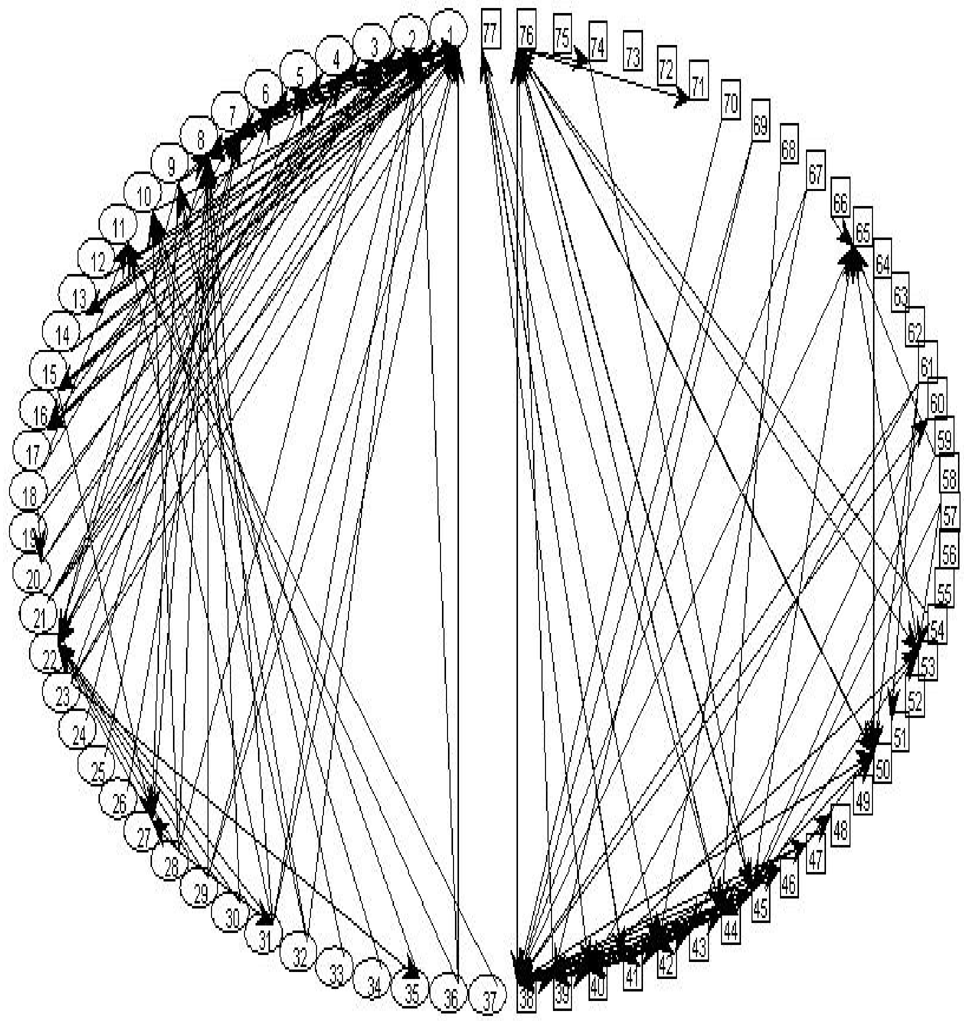

Fig. 1. Intra-community links: (lhs) network of 37 agents (circles) corresponding to the IDP community, i.e. the C_0 matrix; (rhs) network of 40 agents (squares) corresponding to the DED community, i.e.the D_0 matrix. In each community there are several nodes/agents not linked to its community: 5, 11, 12, 14, 17, 18, 19, 21, 23, 24, 25, 26, 33, 34, 37, and 48, 49, 52, 54, 55, 56, 57, 59, 61, 62, 63, 64, 67, 68, 69, 70, 72, 73, 75 respectively. An arrow starting from j and pointing to i is drawn if $m_{ij} = 1$ and $m_{ji} = 0$, for so called directed links (DL)

3.1 Degree distributions

In this sub-section we report the results of the analysis of the empirical distributions of in-degree and out-degree, in order to be testing the hypothesis of

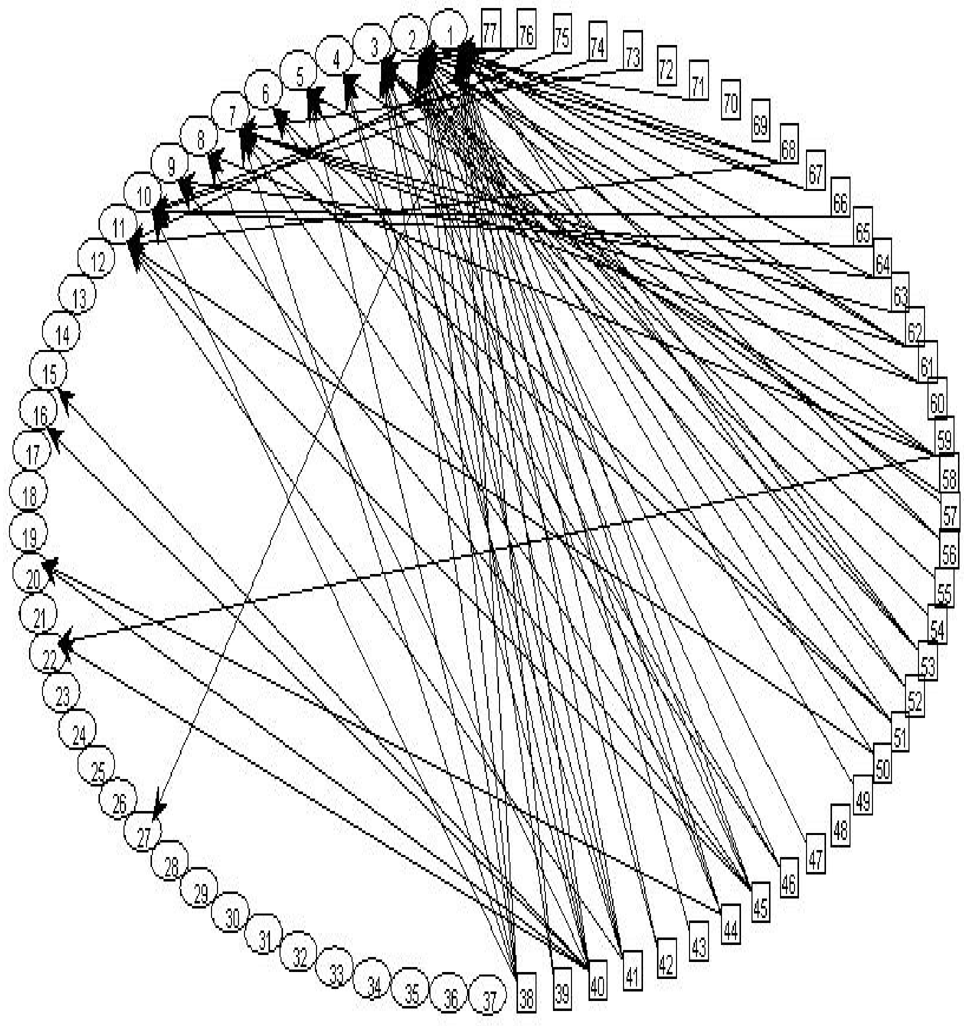

Fig. 2. Network corresponding to off-diagonal block rectangular matrix A, i.e. when some DED cites some IDP. An arrow indicates a directed link (DL)

power law (expected for "scale free networks") against exponential (expected for "random networks") distribution estimated on both probability density and distribution of links. Notice that a power law or exponential law, if found, might also be meaningful for detecting the kind of growth of the network, respectively through a preferential attachment mechanism or through a random one. One may also neither know *a priori* whether different behaviors exist or not for the out- and in-degree distribution, - moreover depending on the matrix of interest. Fig. 5 shows, for each node, the out-degree and the in-degree, respectively, i.e. the number of links exiting from and entering into each node. Fig. 6 reports the out- and in-degree histograms corresponding to the various

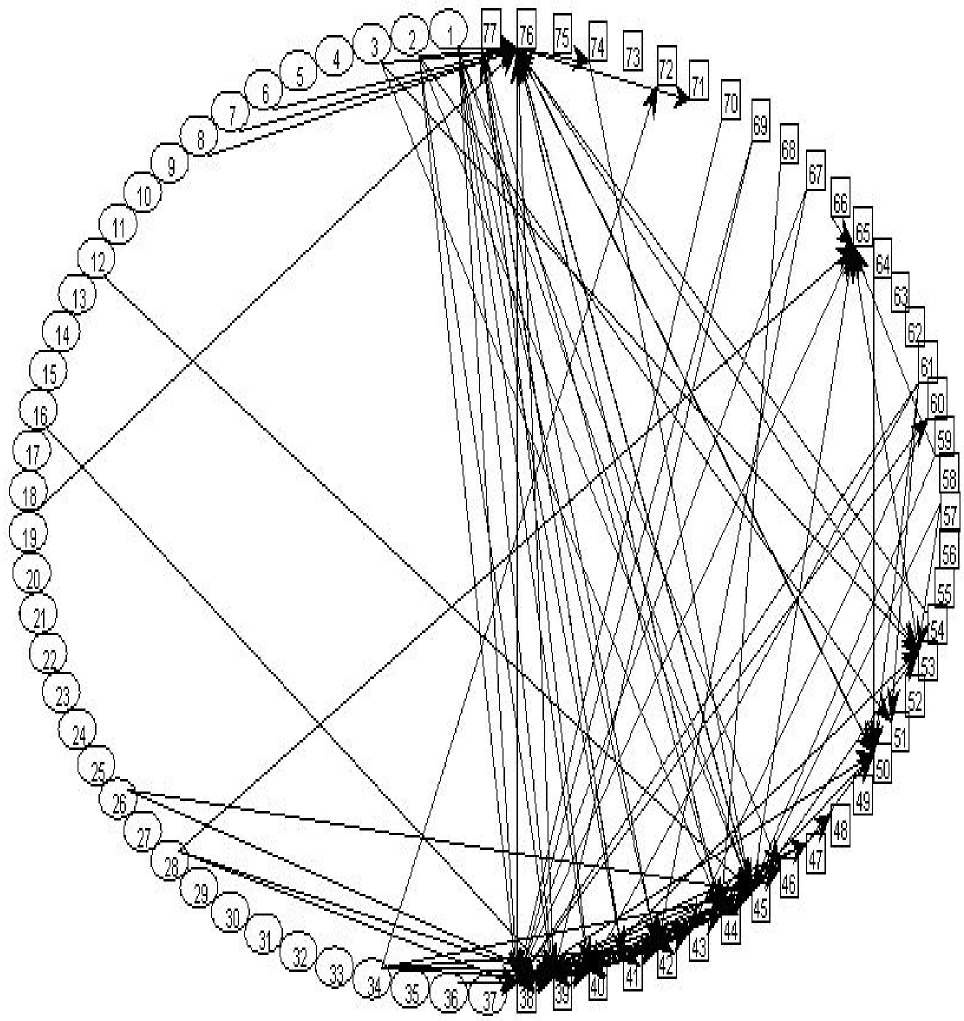

Fig. 3. Network corresponding to off-diagonal block rectangular matrix B, i.e. when some IDP cites some DED. An arrow indicates a directed link (DL)

matrices. Finally notice that in doing so we add a quantitative set of values for an answer to a question raised in [41] on the classes of SWN so far examined in the literature.

A power law behavior is searched through a log-log plot method, i.e. the best fit of a power law to the empirical density $p(x)$ of in-degree and out-degree for each matrix

$$p(x) \sim x^{-b} \tag{4}$$

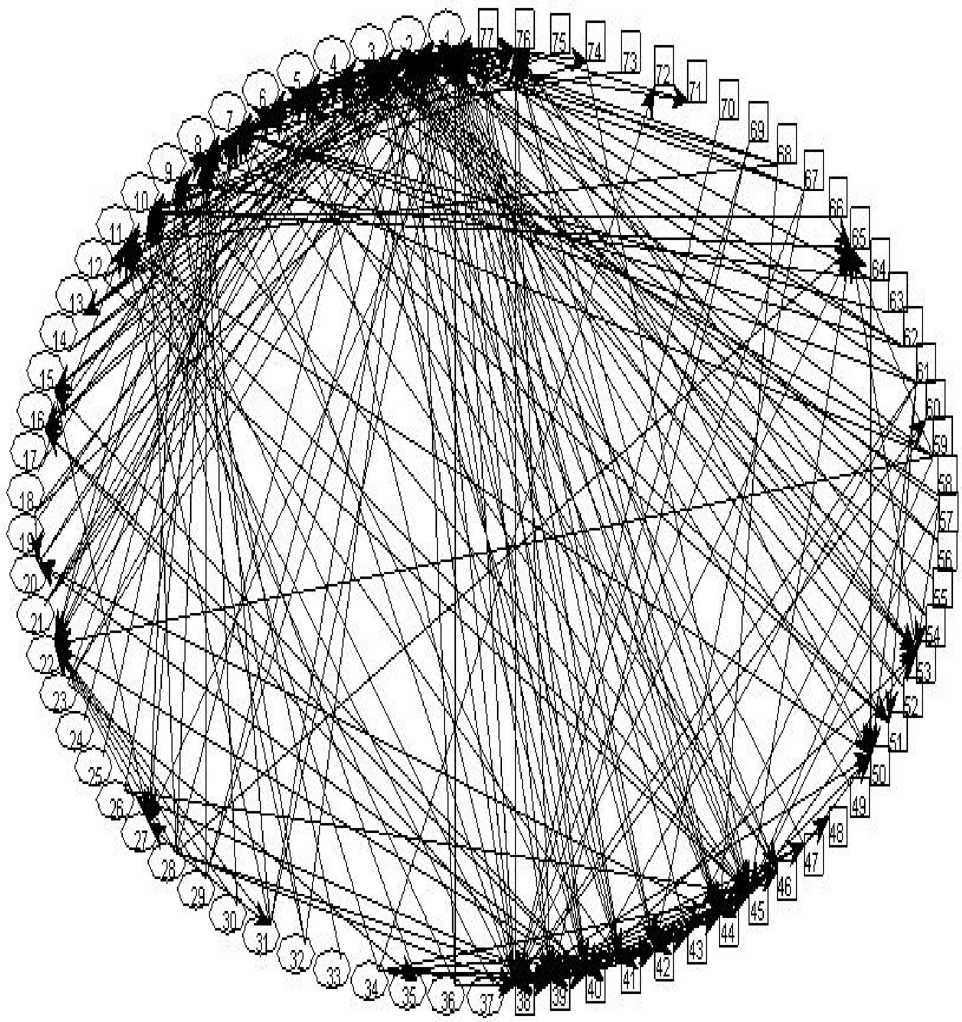

Fig. 4. Network corresponding to the whole community described by the M_0 matrix. An arrow indicates a directed link (DL). In contrast to Fig.1 no node is isolated

where x refers to the degree of a node, i.e. the number of links of the node. The exponential law behavior is searched through the best fit of

$$p(x) \sim e^{-bx} \tag{5}$$

to the empirical density $p(x)$ of out-degree and in-degree for each matrix.

A gaussianity test of the residuals has been performed for cross-checking the goodness of the fits through the Jarque-Bera (JB) test [42]. The test returns the logical value 1 if it rejects the null hypothesis at the 5 % significance level,

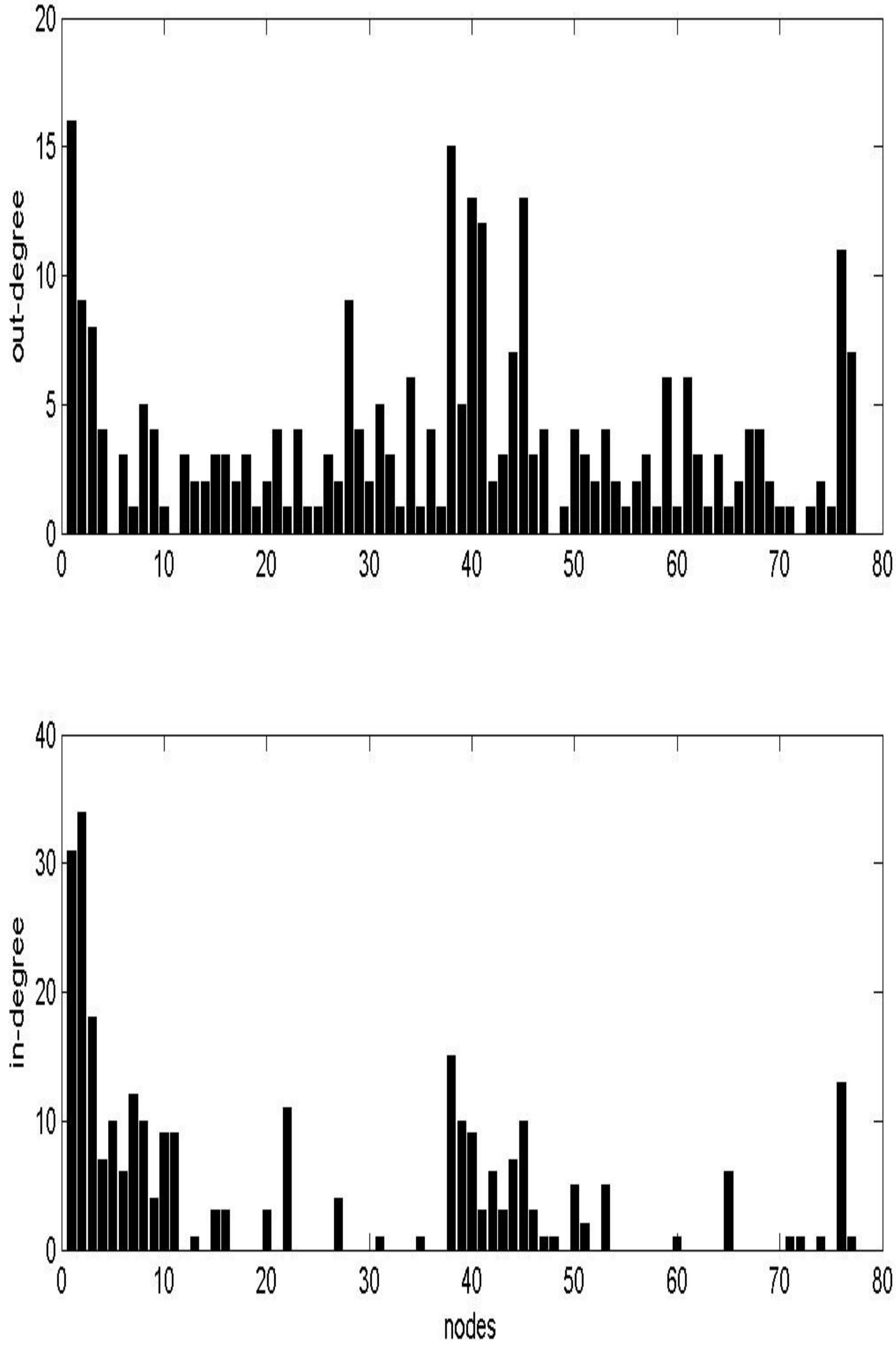

Fig. 5. Out−degree (top) and in-degree (bottom) for each node, from Matrix M_0.

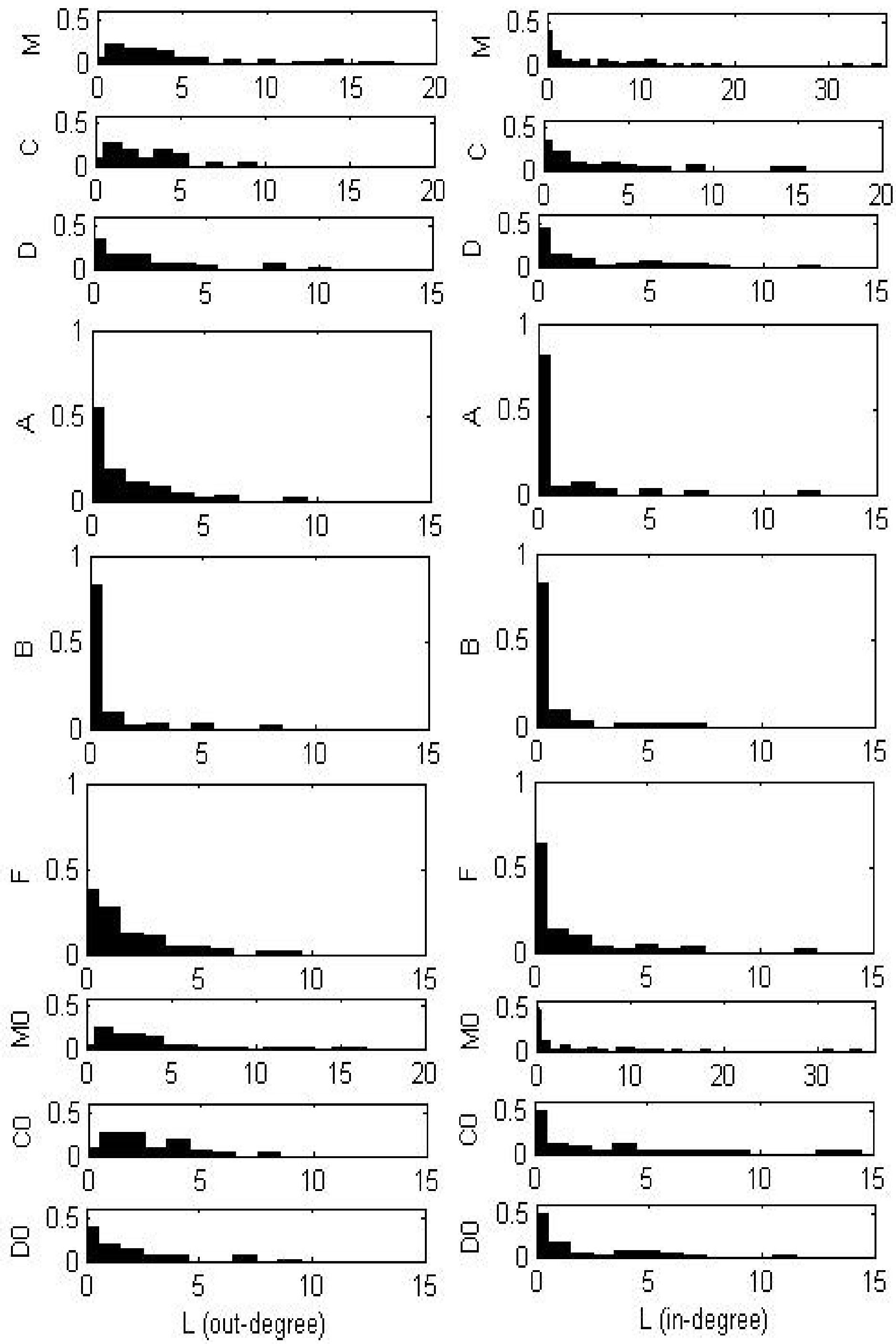

Fig. 6. Histogram of the number L of out−degrees and in−degrees for each matrix recalled in Table 1

and 0 if it does not. It can be concluded that the each fits is statistically meaningful when the Jarque-Bera (JB) test confirms the Gaussianity of the residuals of *each* fit in each case within the 95% confidence bounds. Table 2 reports the values of the parameter b best fits for the power law and exponential law respectively for the empirical densities of out-degrees and in-degrees for each matrix/network of interest.

The behavior of the out-degree density is described *quasi* equally well by a power law or by an exponential. The scale-free exponent of the out-degree of M and its subnetwok is $b \sim 1.0$ quite far from the scale free exponent of other communities, like the network of actors ($b = 2.3$), or citation networks ($b \sim 3$) (see data compendium in [43]). So mechanisms for out-links preferential attachment of IDP-DED network, if any, are less significant than the one shown in other networks. The more so for the b exponent close to $b = 0.5$ for C and C_0, for the out-degree density. One might thus recognize that the preferential attachment is much less likely than for M and also D, or M_0 and also D_0, that show a higher value of b. However a value $b \sim 4/3$ is found for the off-diagonal matrices A and B. This can be interpreted as due to the fact that authors of different groups most refer to the most famous one of the other group, adding the strength of the outstanding author if compared to the others of his group.

The comparison of the power law exponent of the in-degree cannot be performed because the JB test rejects the Gaussianity of the residuals. Disregarding the JB test and accepting the exponent b values there are found either close to 1.0 or 4/3. This would indicate a preference to citing a small group of authors for the in-degree density.

The exponential law describes the in-degree decay much better than the power law. In fact, it is seen that the JB test implies to reject the hypothesis of the power law behavior for the in-degree (Table 2). The power law is accepted only for M, for which also the exponential decay is accepted. We can only conclude that in-degree of M has a fast decay that is "between" the power law and the exponential. The prevailence of the exponential decay only signals that the small-world hypothesis may hold, but the rejection of the power law surely means that the network is not scale free, and that mechanisms of network growth as through preferential attachment are unlikely to occur.

There are similarities in the range and hierarchy of values for the decay rate parameter b of the out- and in-degree densities for a given matrix. However a large difference is found in the relative values: a factor of two frequently occurs. The most marked difference is for the matrix C and C_0, having $b \sim 0.15$ and $b \geq 0.75$ for the out- and in-degree respectively. Observe that the fact that the DED are more prone to cite themselves than the IDP much influences this b value: it is very high $b \sim 1.0$ for the former, very low for the latter $b \sim 0.5$.

	out-degree		out-degree		in-degree		in-degree	
	b (power law)	JB	b (exp. law)	JB	b (power law)	JB	b (exp. law)	JB
M	1.16 (0.88,1.44)	1	0.13 (0.03,0.22)	0	0.72 (0.48,0.97)	1	0.76 (0.54,0.98)	0
C	1.03 (0.21,1.85)	0	0.12 (-0.11,0.34)	0	0.70 (0.38,1.01)	0	0.50 (0.33,0.68)	0
D	0.75 (0.26,1.23)	0	0.39 (0.21,0.57)	0	0.64 (0.15,1.13)	0	0.83 (0.47,1.20)	0
A	1.31 (0.74,1.88)	0	0.88 (0.65,1.11)	1	0.59 (0.06,1.12)	0	2.83 (1.00,4.67)	0
B	0.68 (-0.62,1.99)	0	2.20 (1.63,2.76)	0	1.00 (0.52,1.47)	0	2.17 (1.81,2.54)	0
F	1.44 (1.14,1.74)	0	0.48 (0.38,0.58)	0	0.96 (0.32,1.60)	0	1.35 (0.96,1.75)	0
M_0	1.26 (0.93,1.58)	1	0.16 (0.03,0.29)	0	0.53 (0.20,0.86)	0	1.26 (0.84,1.69)	0
C_0	1.22 (0.39,2.06)	0	0.15 (-0.15,0.44)	0	0.57 (0.19,0.95)	1	1.17 (0.61,1.74)	1
D_0	0.80 (0.27,1.33)	0	0.48 (0.27,0.69)	0	0.63 (-0.03,1.29)	0	0.99 (0.54,1.44)	0

Table 2
Test of power law and of exponential decay for the empirical densities of out-degree and in-degree distributions for each discussed matrix. The columns labelled JB report the results of the Jarque-Bera test, i.e. the test returns the logical value 1 if null hypothesis is rejected, and 0 if the null hypothesis is accepted. The standard deviation confidence interval is given in parentheses in each b case

This indicates either a marked narcistic or ego effect of the former community members, or an attitude in citing only respected or respectable scientists as in usual scientific publications [44,45]. This fast decay has an interpretation however: it arises from the fact that there are very few persons citing or being cited by many others, see the piling at low value of the degree on Fig. 6.

As a conclusion, let it be stated that we hardly observe any clear pattern of acceptance/rejection of either empirical laws, nor thus on the type of network kinetics. In fact, laws with tails should be considered to be very rough approximations of distribution functions when the finite size of the system is so much marked.

3.2 Network Assortativity

In order to indicate some aspect of the attachment process in a network, one can calculate its so called assortivity. The term assortativity is commonly used after [36,37] to refer to a preference for a network node to be attached to others depending on one out of many node properties. Assortativity is most often measured after a (Pearson) node degree correlation coefficient $r \in [-1,1]$, Eq.(6) below, such that $r = 1$ indicates perfect assortativity, $r = -1$ indicates perfect disassortativity, i.e. perfect negative correlation. Let p_i^{out}

	A	B	D_0	M	M_0	F	D	C_0	C
r	0.253	0.397	0.451	0.460	0.461	0.486	0.534	0.621	0.644

Table 3
Value of the assortativity coefficient Eq.(6) following Eq.(26) of Newman [37]. The relevant matrices are ranked hierarchily, from left (low r value) to right (high r value) indicating some increasing preferential attachment

(p_i^{in}) be the probability that a randomly chosen vertex i will have out-degree (in-degree) k_i^{out} (k_i^{in}): they can be obtained/read from Fig. 6. Let N be the number of nodes of the network, L the number of links, and m_{ij} the adjacency matrix elements. By definition, adapting to our notations Eq.(26) of [37], the (network) assortativity coefficient r is given by:

$$r = \frac{\sum_{i,j=1}^{N} q_i^{in} q_j^{out} m_{ij} - (\sum_{i,j=1}^{N} q_i^{in} m_{ij})(\sum_{i,j=1}^{N} q_j^{out} m_{ij})/L}{\sqrt{[\sum_{i,j=1}^{N} (q_i^{in} m_{ij})^2 - (\sum_{i,j=1}^{N} q_i^{in} m_{ij})^2/L][\sum_{i,j=1}^{N} (q_j^{out} m_{ij})^2 - (\sum_{i,j=1}^{N} q_j^{out} m_{ij})^2/L]}} \quad (6)$$

where

$$q_i^{out} = \frac{k_i^{out} p_i^{out}}{\sum_i k_i^{out} p_i^{out}};$$

and

$$q_i^{in} = \frac{k_i^{in} p_i^{in}}{\sum_i k_i^{in} p_i^{in}}$$

.

For the networks of interest here, we have found, see Table 3, quite positive values for the *assortativity*, ranging between 0.25 and 0.65. Again one can observe that the self-citations influence the r value by 17% in the case of DED. The $0 \le r \le 1$ values reflect the fact that there is a small number of (to be later further precised as being more famous) authors either largely citing each other, or largely cited by each other, while the others show a low activity of citing. This is the case for the DED, - matrix D. The r value for the whole network (matrix M or M_0 is approximately the same as for the IDP, - matrix C or C_0. Notice that A and B alone have the least assortative feature than the other matrices (networks), as could be expected from a psychological view point on this matter: the citations between rival groups tend to be biased, i.e. they aim at enhancing the most relevant authors/leaders of the rival community only [34,44,45].

3.3 Clustering

The tendency of the network nodes to form local interconnected groups is usually quantified by a measure referred to as the clustering coefficient [17].

The amount of studies on this characteristic of networks has led to the particularization of the term to focus on different features of the networks. Here we consider the global clustering coefficient and the local clustering coefficient.

Indeed, the most relevant elements of a heterogeneous agent interaction network can be identified by analyzing the global and local connectivity properties. In the present case those individuals leading the opinion of IDP and DED groups can be attempted to be identified by analyzing the number of triangles with nodes belonging to the same community, so called homogeneous triangles, or not, so called inhomogeneous triangles, beside the type of (directed or/and undirected) links of their citation network. The former number gives some hierarchy information, the latter some reciprocity, i.e. recognition of leadership or proves of some challenging, conflict, in time.

3.3.1 Global clustering coefficient

The global clustering coefficient (GCC) *of the network* is defined as the average of c_i *over all the vertices* in the network, $< c >= \sum c_i/N$, where N is the number of nodes of the network, and the clustering coefficient c_i of a vertex i is given by the ratio between the number e_i of triangles sharing that vertex, and the maximum number of triangles that the vertex could have. If a node i has k_i neighbors, then a clique, i.e. a complete graph, would have $k_i(k_i-1)/2$ triangles at most, thus $c_i = 2e_i/k_i(k_i-1)$.

Whence one starts by considering the various triangles on the networks and next proceeds in calculating the clustering coefficient. This fact should lead to obtain two pools containing respectively the most important opinion leaders in the IDP and DED groups. However one has to generalize the 16 triplets nomenclature found in the literature, [51] in order to consider the bi-community nature of the IDP-DED network, i.e. the node tag itself, beside the types of links, i.e. UL or DL, in between when they exist.

This leads to consider a set of 104 different possible types of triplets (triangles if three nodes are linked) shown in Figs. 10-11 in Fig. 8 with appropriate notations emphasizing the number of C and D nodes beside the number of DL and UL [1]. The counting result is reported in Table 4.

[1] Due to [51] which listed triplets according to the number of UL and DL, the case #11 occurs when two nodes are connected by two UL, thus have 4 links, - a mere triangle which has (at least) 3 links thus gets a lower # in [51]. Thus triangles are found in patterns (9, 10) and (12, 13, 14, 15, 16). We have kept the [51] order in our generalization. We apologize to the reader for this apparently regrettable incoherence stemming from a different point of view between [51] and our aim. A similar comment on misfit nomenclature can be made if one wishes to use the rank list of (partially or not) connected vertices in triplet cluster in [52]

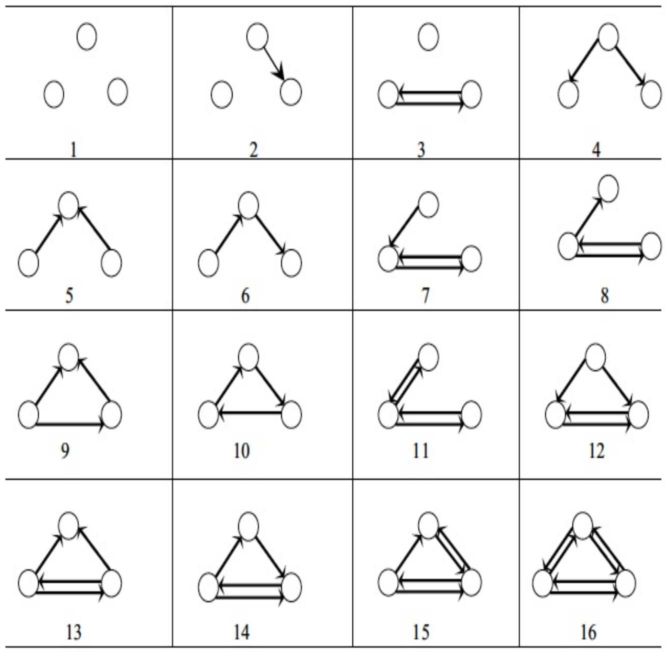

Fig. 7. Pajek convention [51] Fig. 15, p. 52 for labelling 16 triplets of nodes variously linked.

Notice at once that triangles containing elements/nodes from *two* groups are found to be the most abundant: for the simplest triangle; #2C (=26) +#1C (=33) $\geq$ #3C (=33) + #0C (=12). Over all: there are 159 hybrid triangles, i.e. containing members from rival communities, for only 71 "homogeneous" triangles. On the other hand several cases are completely missing: see many 0 values in Table 4. Notice also that the number of triangles indicating DL *transitivity* [2], like #9 are the most numerous ones, far more numerous than those indicating a "round-and-around" $attack - citation$ pattern, i.e. #10. There are even non-existent when heterogeneous triangles are considered. This *a priori* unexpected fact immediately reflects a strongly peculiar interaction between both opinion groups. However when UL occur, the transitivity and attack-citation patterns are difficult to disentangle. Cases #15 and #14 are surely the most abundant ones in this case.

Unfortunately, one cannot easily explain from values on both #15 and #14 (triplet type) data lines whether there is some agent *support* by its own community or some arguing by one or two of the same community against the other along a similar line of arguments. In fact the measurements outlined

[2] Let us use the symbol $\rightarrow$ for representing citations, i.e. $j \rightarrow i$ means that j cites i. Let us assume that $j \rightarrow i$ and $i \rightarrow k$. Then the relationship represented by $\rightarrow$ indicates a so called transitivity one if $j \rightarrow k$

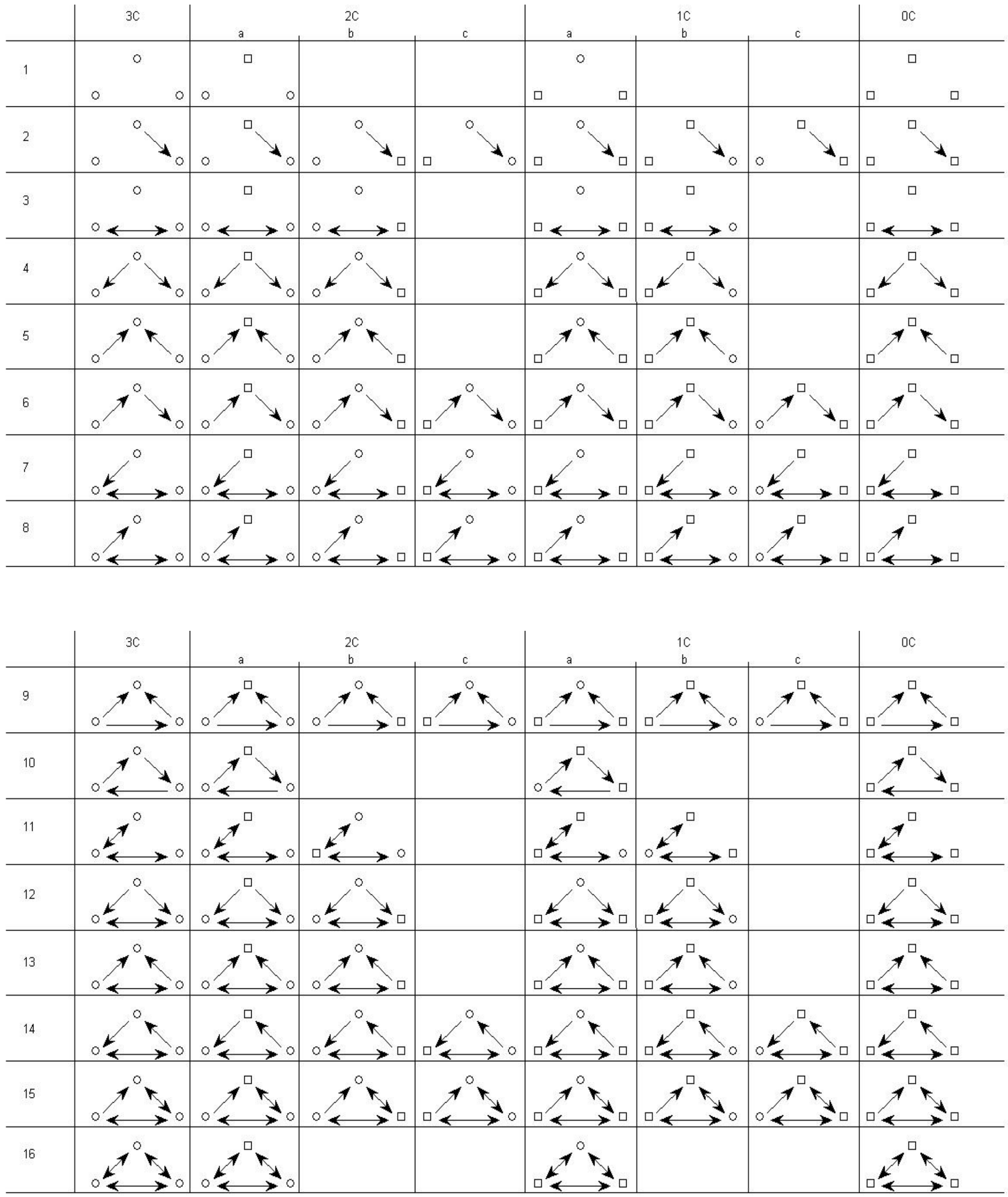

Fig. 8. Present convention for labelling 104 triplets (triangles if three nodes are linked) of nodes variously linked, taking into account the node nature: a circle for *creationists* (C); a square for *evolutionists*. The nomenclature of triangles with tagged vertices/nodes and biased edges/links has been generalized from the 16 combinations shown from the Pajek software [51], that only consider the configurations shown in the first column. Such a generalization is necessary when nodes, tagged like 1/2-Ising spins, belong to one out of two communities. Table 4 reports the triplets/triangles counted on the present case of IDP-DED opinions.

triplet type		3C	2C	2Ca	2Cb	2Cc	1C	1Ca	1Cb	1Cc	0C
1	57376	5320	20631	20631	-	-	23533	23533	-	-	7892
2	11759	1874	4553	493	1708	2352	3888	1853	524	1511	1444
3	1315	103	397	118	279	-	563	267	296	-	252
4	240	40	75	47	28	-	91	8	83	-	34
5	963	185	453	15	438	-	267	218	49	-	58
6	511	118	171	30	22	119	149	19	91	39	73
7	489	52	206	65	121	20	186	33	20	133	45
8	171	19	54	4	33	17	57	37	15	5	41
9	104	33	26	1	6	19	33	29	0	4	12
10	2	1	0	0	-	-	0	0	-	-	1
11	50	7	24	24	0	-	16	2	14	-	3
12	13	0	7	0	7	-	6	2	4	-	0
13	16	0	5	0	5	-	6	6	0	-	5
14	34	2	9	1	4	4	18	3	0	15	5
15	42	2	15	1	1	13	18	9	9	0	7
16	19	1	5	5	-	-	11	11	-	-	2

Table 4

Column 1 lists the type of (connected or not) triplets as conventionally labelled in the pajek software [51] and recalled in Fig. 7 . Column 2 gives their number for the whole network of IDP-DED studied here. The numbers in the other columns give the number of respective cases according to the notations in Fig. 8 . Recall that triangles may include several different C and D agents: columns 3C, 2C, 1C, 0C, respectively refer to a triangle with 3 creationists, 2 creationists (and one darwinian evolutionist), 1 creationist (and 2 darwinian evolutionists), and 0 creationist (and 3 darwinian evolutionists). The letters (a,b,c) serve to distinguish among the different configurations shown in Fig. 8; the order is arbitrary.

till now cannot shed light on the relevance of a particular individual to any network structure, because they provide a collective measure, whence do not focus on each single author. This emphasises the need or interest of looking at the A and B matrices more than at those on the diagonal, i.e. C and D, as already proclaimed here above.

The detection of such triangles is at the base of our estimate of the global clustering coefficient Γ, defined as the ratio between the number of triangles and the total number of possible triplets, both numbers which can be easily

[51] notations	total	3C	2C	1C	0C
Γ	0.0867	0.0848	0.0638	0.1072	0.1119

Table 5
Clustering coefficient, i.e. the ratio of the number of configurations of triangles, i.e. from #9 to #16 (excluding the #11th) to the number of configurations implying at least three different nodes and two links, i.e. #4 to #16 in Fig. 7.

obtained by summing the corresponding values listed in Table 4.

Notice that Γ can only be unambiguously calculated for the as defined in [51] configurations; otherwise a redistribution of the configurations has to be made to take into account symmetry and chirality conditions among the possible104 configurations.

3.3.2 Local clustering coefficient

In the literature [34], the term 'clustering coefficient' refers also to other quantities, relevant to understand the way in which nodes form communities, under some criterion. By definition, the local clustering coefficient (LCC) Γ_i for a node i is the number of links between the vertices within the nearest neighbourhood of i divided by the number of links that could possibly exist between them. It is relevant to note that the GCC is not trivially related to the LCC, e.g. the GCC is not the mean of LCC. In the former case, triangles having common edges are emphasized, in the latter case only the links; the number of links common to triangles can vary much with the number of connected nearest neighbour nodes indeed. Basically, the latter quantifies how its neighbors are close to being part of a complete graph. LCC rather serves to determine whether a network is a SWN [17] or not.

For a directed graph, for each neighbourhood of node i, there are at most $k_i(k_i-1)$ links that could exist among the vertices within the neighbourhood, - where k_i is the total (in- plus out-) degree of the vertex, i.e. $k_i = \sum_{j\neq i}(m_{ij} + m_{ji})$. Therefore [3]

$$\Gamma_i = \frac{k_i(k_i-1)}{\sum_{j=1}^{N}(m_{ij}+m_{ji}) + \sum_{k=j+1}^{N} \Upsilon(m_{ij}+m_{ji})\Upsilon(m_{ki}+m_{ik})(m_{jk}+m_{kj})} \quad (7)$$

where Υ is the *sign* function, i.e. $\Upsilon(x) = 0$ if $x = 0$; $\Upsilon(x) = 1$ if $x > 0$; $\Upsilon(x) = -1$ if $x < 0$. The first term in the denominator counts the links among i and its neighbors. The second term in the denominator counts the

[3] In the summations in the denominator of Eq.(7), and similarly for Eq.(8) and Eq. (9), we have not indicated the restriction $j \neq i$, $i \neq k$ and $k \neq j$ because we consider the M_0, C_0, and D_0 matrices only

	M_0	C_0	D_0	A	B	F
$\bar{\Gamma}$	0.411	0.387	0.321	0.216	0.124	0.245
$max_i \Gamma_i$	0.750	0.667	0.667	0.500	0.500	0.500
$min_i \Gamma_i$	0.117	0.140	0	0	0	0
$\bar{\Gamma}^s$	0.281	0.301	0.301	0.211	0.206	0.238
$max_{i,s} \Gamma_i^s$	1.0000	1.0000	1.0000	0.5000	0.5000	1.0000
$min_{i,s} \Gamma_i^s$	0	0	0	0	0	0

Table 6
Local clustering coefficient, rounded to their third decimal, of the relevant matrices calculated through Eq.(7) and $\bar{\Gamma} = \frac{1}{N} \sum_i \Gamma_i$, where the sum is taken on the nodes of the network, and N is the number of nodes. Values are compared with the maximum and the minimum on the set of nodes. The bottom half of the table reports $\bar{\Gamma}^s$, i. e. the mean of the value $\bar{\Gamma}$ calculated on 1000 matrices obtained from the previous ones after shuffling. $max_{i,s} \Gamma_i^s$ and $min_{i,s} \Gamma_i^s$ are, respectively, the maximum and the minimum taken over each Γ_i of each simulation. Zeros are due to the fact that the shuffle may lead to isolated nodes.

links among the neighbors of i. The lower limit for the index k in $\sum_{k=j+1}^{N}$ is introduced for avoiding double counting.

Considering only the links *exiting* i, one could define a similar quantity as follows

$$\Gamma_i^{out} = \frac{k_i(k_i - 1)}{\sum_{j=1}^{N}(m_{ji}) + \sum_{k=j+1}^{N} \Upsilon(m_{ji})\Upsilon(m_{ki})(m_{jk} + m_{kj})} \tag{8}$$

or considering only the links *entering* i as follows

$$\Gamma_i^{in} = \frac{k_i(k_i - 1)}{\sum_{j=1}^{N}(m_{ij}) + \sum_{k=j+1}^{N} \Upsilon(m_{ij})\Upsilon(m_{ik})(m_{jk} + m_{kj})} \tag{9}$$

However, the Γ_i^{out} and Γ_i^{in} plots are not shown for space saving.

Fig. 9 shows the LCC Γ_i, both reporting the list of nodes on the x-axis and the same list sorted out according to the decreasing order of Γ_i, while Fig. 10 is the histogram of Γ_i. These plots show that there exists a large set of nodes having a Γ_i with zero value, and others, a very large number of triangles, mainly having a value close to 1/3 or 1/5. Recall that the lower the Γ value, the less "fully connected" appears the network; such is the case of the intra-community matrices, A and B and somewhat surprisingly D_0.

The local clustering coefficient $\bar{\Gamma}$ for the whole system is given as the average of the local clustering coefficient Γ_i over all the nodes [17], i.e. $\bar{\Gamma} = \frac{1}{N} \sum_i \Gamma_i$,

where N is the number of nodes. The mean values for the relevant matrices, as well as the range through the min and max values, are reported in Table 6.

Recall also here that a graph is considered *small-world*, if its average local clustering coefficient is significantly **higher** than a random graph constructed on the same vertex set. Therefore this average local clustering coefficient can be usefully compared with the mean value obtained by randomizing the network and its subnetworks. Table 6 shows the results over 1000 random permutation of network links. The matrix M has been shuffled before extracting again from it each submatrix (M_0, C_0, D_0, A, B, F). For each selected matrix, let $\bar{\Gamma}(n)$ the value $\bar{\Gamma}$ calculated after the $n-th$ shuffle. Then $\bar{\Gamma}^s = \frac{1}{1000}\sum_n \bar{\Gamma}(n)$. The values reported in Table 6 correspond to an average over 1000 cases of the respective Γ.

It is seen that the mean of $\bar{\Gamma}$ of M_0 (0.41) much higher than any of the mean of its random counterparts (0.28), thereby indicating that the network is very far from a random graph, while the values inside each group are more close. This means that the intra-group interaction is quite close to a random one, but the build-up of inter-group relationship is far from being random. Thus we confirm that the present networks look like SWN rather than RN.

3.4 Average Overlap Index

Finally for characterizing members of communities, in another hierarchical way, let us also calculate the Average Overlap Index (AOI) O_{ij}; its mathematical formulation and its properties are found in [38] in the case of a unweighted network made of N nodes linked by (ij) edges,

$$O_{ij} = \frac{N_{ij}\,(k_i + k_j)}{4\,(N-1)\,(N-2)}, \qquad i \neq j \tag{10}$$

as before excluding self-citation loops in calculating k_i, and where N_{ij} is the measure of the common number of neighbors to the i and j nodes. N.B., in a fully connected network, $N_{ij} = N - 2$. Of course, $O_{ii} = 0$ by definition.

The *Average Overlap Index* for the node i is defined as

$$\langle O_i \rangle = \frac{1}{N-1}\sum_{j=1}^{N} O_{ij}. \tag{11}$$

This measure, $\langle O_i \rangle$, can be interpreted indeed as an other form of clustering attachment measure: the higher the number of nearest neighbors, the higher the $\langle O_i \rangle$, the more so if the i node has a high degree. Since the summation is

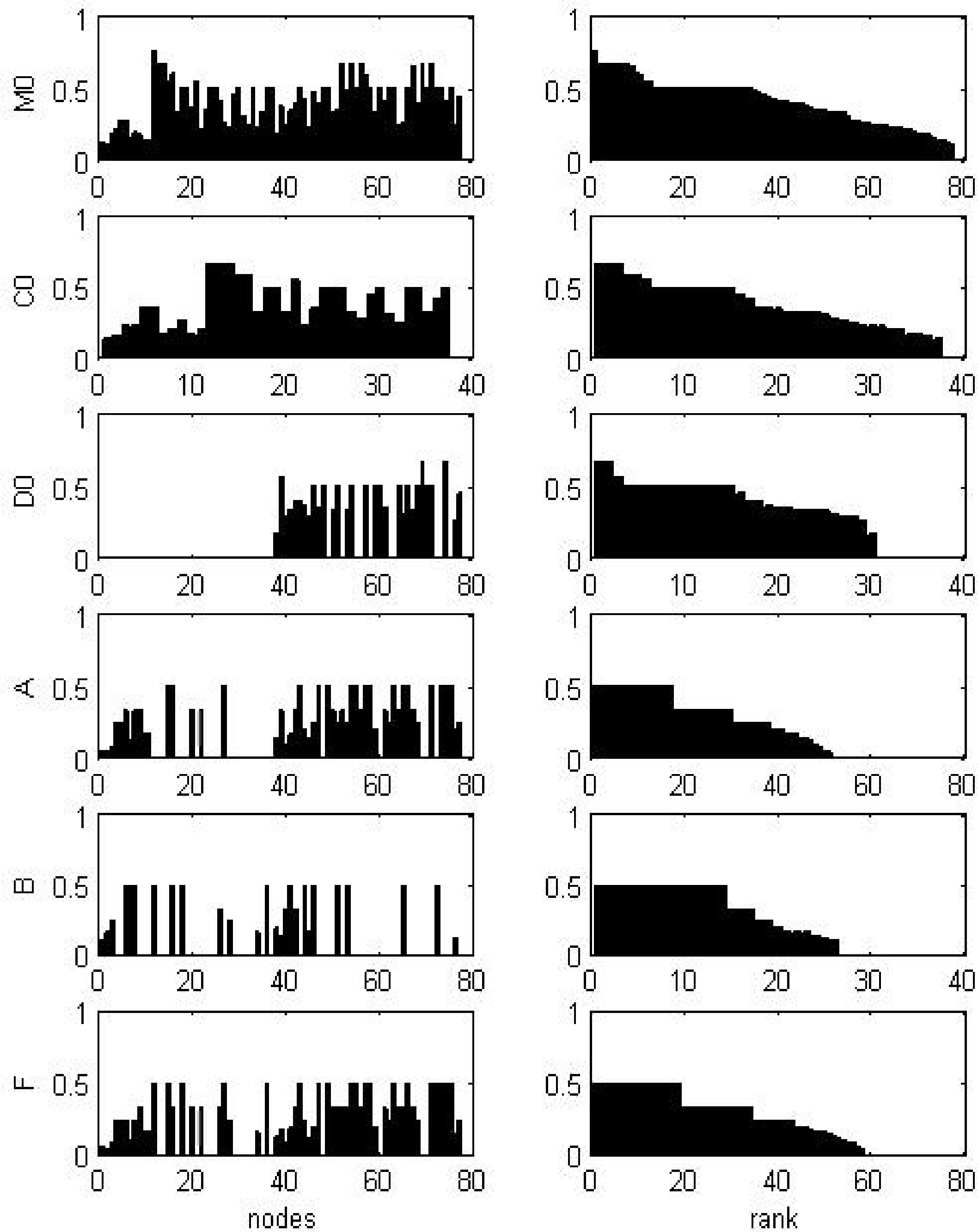

Fig. 9. Local clustering coefficients (LCC) Γ_i as obtained from the M_0, C_0, D_0, A, B, F matrices: (lhs): on the x-axis is the node number; (rhs): the nodes ranked according to the decreasing value of LCC on the x-axis

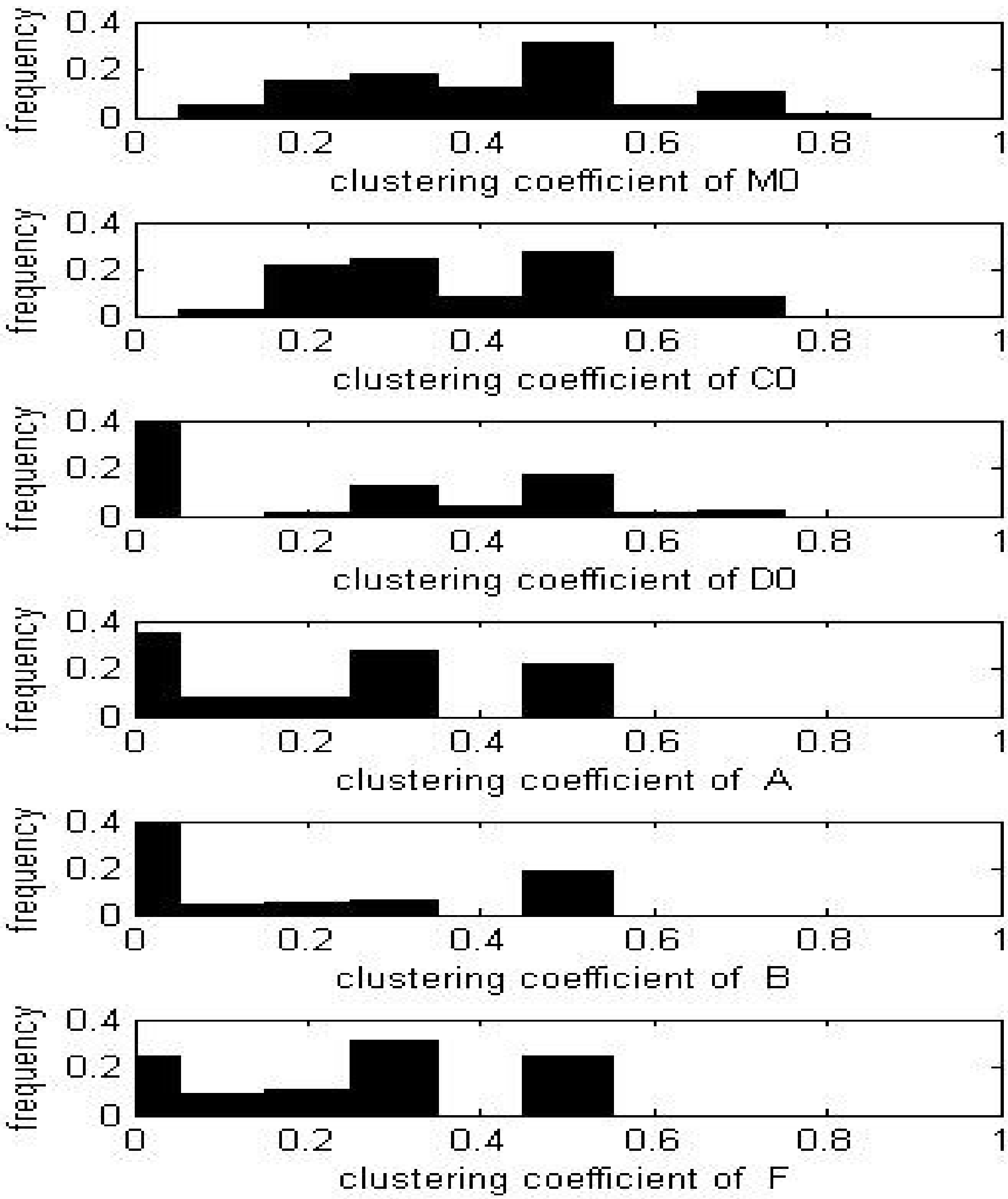

Fig. 10. Histograms of the local clustering coefficients (LCC) Γ_i as obtained from the M_0, C_0, D_0, A, B, F matrices respectively (top to bottom).

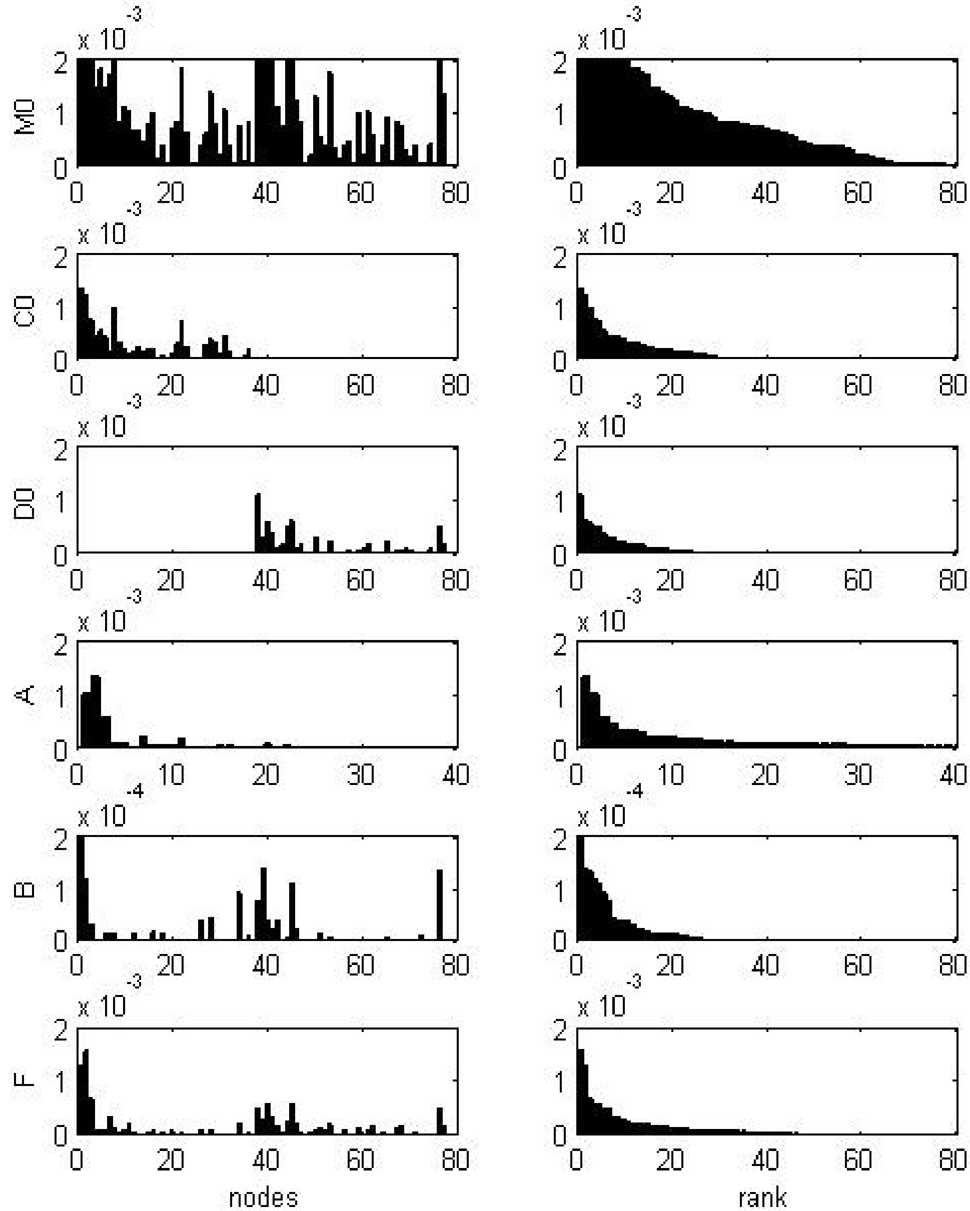

Fig. 11. Average overlap index (AOI) as obtained from the M_0, C_0, D_0, A, B, F matrices: (lhs): on the x-axis is the node number; (rhs): on the x-axis, the nodes are ranked according to the decreasing AOI value

made over all possible j sites connected to i (over all sites in a fully connected graph), $\langle O_i \rangle$ expresses a measure of the local density near the i site. In magnetism, the links are like exchange integrals between spins located at i and j. That average over the exchange integrals is a measure of the critical (Curie) temperature at which a spin system undergoes an order-disorder transition.

agent name	node number	AOI x 10^3 value
M.Behe	2	110
W.Dembski	6	108
S.C.Meyer	3	54
C.R.Thaxton	8	33
.	.	.
J.Murrel	33	0
R.Pennock	38	69
B.Forrest	40	55
E.C.Scott	45	52
R.Dawkins	76	46
E.Sober	44	35
W.Elsberry	41	32
K.Miller	39	28
.	.	.
M.Singham	72	0

Table 7
IDP and DED agents ranked according to their AOI; we kept only those having an AOI greater than 0.025 and the least one for illustration

Therefore $\langle O_i \rangle$ can also be interpreted as a measure of the stability of the node versus perturbations due to a thermal cause. Thus, here, a high $\langle O_i \rangle$ value reflects the i node strong attachment to its community.

The *average overlap index* of each node, obtained according to Eq.(11), depending on the matrix of interest are given in Fig. 11. The order of magnitude of the $\langle O_i \rangle$ values are $\sim 10^{-3}$, much smaller than in other investigated cases, like in [38] or [53]. This is due to the low value of N_{ij}, somewhat reflecting the low GCC value for the whole network, i.e. 0.0867, in Table 5., and the LCC values in Table 6. Whence, a rough estimation suggests that $\Gamma_i/N \ \sim O_i \sim 10^{-3}$, in good agreeemt indeed.

The AOI names and list in Table 7 can be compared to the results on the number of triangles of the most relevant actors. It was noticed in [28] that the IDP leaders had more homogeneous triangles (type #3C in the notations of Table 4 and Figs.7-8) than others, while the DED had (somewhat surprisingly) no triangle of type #0C, but IDP and DED agents had many "inhomegeneous"

triangles. They are

- *M.Behe*, *W.Dembski*, *S.C.Meyer*, and *C.R.Thaxton*;
- *R.Pennock*, *B.Forrest*, *E.C.Scott*, *R.Dawkins*, *E.Sober*, *W.Elsberry*, and *K.Miller*.

respectively.

4 Conclusions

In primordial science, proto to scientific crises, there are inter-connections between distinct disciplines which induce a link, between authors intra-connected otherwise through their discipline, i.e. communities on networks. This is also displayed in para-scientific disciplines, which leads to the temptation of assimilating either science to philosophy, metaphysics, religion or the opposite. A debate attitude leads to a social behavior which has intuitively the same nature as that in well founded scientific disciplines, i.e. a citation pattern to refute ideas or hypotheses, on one hand, and to claim some support from one's own community, on the other hand.

In particular, the creationism proposal and its subsequent sequel of propaganda publications is reminiscent of the diffusion of topics in science. This has incited us into examining this very modern case of so called *scientific community* connectivities from a network of citations point of view. However community detection as in [54,55,56,57] is less relevant here from a statistical physics of network study than the level of exchanges [23,24,25] and its reciprocity through arguing. The members of the communities are clearly belonging to one or another, as established by mere reading of their work, through arguments for and against others. Yet the inter-community, and at a lower level of interest the intra-community, links raise interesting network structural questions.

In order to build the network(s), we have applied the usual method [34], namely we have considered a (citation) network of authors placed at nodes, with a link between them if they cite another's paper. E.g. we have *a priori* discriminated two sub-communities: one for scientists favoring *"darwinian evolution"* and others proposing *"creationism"* and *"intelligent design"* as a scientific alternative. There were for the time of observation (1990-2007) of almost equal size, 37 and 40 agents respectively.

We have constructed the two networks and looked for their interaction through the number of undirected and directed links without weighting them nor discussing their chronological sequence. In so doing it is found that the total

number of directed links (219) is much higher than (31) that of undirected links, indicating an intense debate on the controversy messages. There are 119 links between agents belonging to different communities, respectively 86 DL in A, i.e. DED citing IDP but only 33 DL in B, i.e. IDP citing DED. Moreover this number of interconnections (119) is quite superior to the corresponding link number inside the creationists (91) and the evolutionists (71) communities, indicating the importance of the controversy exchanges. Notice that the number of self-citations, in absolute values, is greater (15) for the true scientists than (11) for the pseudo ones, - a known classical sin of that community. The same observation goes true in relative values.

The number of out-degrees is much larger than the number of in-degrees. Their distribution is however not fitting any obvious theoretical law, e.g. the JB test implies to reject the hypothesis of the power law behavior for the in-degree. The exponential decay-like law being more likely indicates that a small-world hypothesis can be imagined, but the rejection of the power law means that the network is not scale free; mechanisms of network growth through preferential attachment are unlikely to be the case.

There are similarities in the range and hierarchy of values for the scale parameter of the out- and in-degree densities for a given network. A fast decay is found also indicating that the networks are not of the random type. This has been confirmed when looking at the clustering coefficients resulting from shuffled adjacency matrices and similarly grouping the nodes into two communities in Sec. 3.3.2. Amaral et al. [41] have proposed three classes of SWN: (i) scale-free networks, characterized by a vertex connectivity distribution that decays as a power law; (ii) broad-scale networks, characterized by a connectivity distribution that has a power law regime followed by a sharp cutoff; and (iii) single-scale networks, characterized by a connectivity distribution with a fast decaying tail. The analyses presented in the main text and summarized here above suggest that the IDP-DED networks belong to the the third case.

In order to characterize the necessarily small networks, based on a adjacency matrices, we have calculated a few *specialisation coefficients*. One could consider other quantities of interest for networks [57], but some relevant point resides in the interestingly non symmetry of the citation networks.

The so called assortativity of the network has been examined in order to search whether there is a proof of any preference of an agent attachment to the sub-network of opposite (in contrast to same) opinion. Examining the whole network, the communities and the inter-community links it is found that the agents are neither perfectly assortative nor perfectly disassortative. From the the values of r, in Table 3, it is asserted that the networks are weakly assortative. It is pointed out that the least assortative features are in the inter-community networks, as expected if the citations between rival groups

are biased, i.e. they aim at enhancing the most relevant authors/leaders of the rival community.

In order to characterize in greater detail the intercommunity structure and exchanges we have considered elementary entities made of agents belonging to different communities. The smallest (geometric) cluster to be examined is the triangle. In order to do so we have generalized the usual nomenclatures of triangles in order to take into account a specific tag to the nodes beside the number of links between these tagged nodes.

We have distinguished between closed triangles and triplets, - in which only two neighbors of a node are *not* otherwise connected. Thus we have calculated the global clustering coefficients, searching for the most relevant triangles.It can be noticed that the number of triangles indicating transitivity, like #9, in the nomenclature (Figs. 7-8) is quite higher than the number of those indicating a round-and-around pattern, i.e. like #10 are completely missing. This *a priori* unexpected fact immediately reflects a strongly peculiar, very direct interaction between both opinion groups, i.e. a weakly collective dynamics. The transitivity patterns being more numerous in #1C than in #2C cases seem also to indicate more agent support by its own (DED) community or some arguing into a similar line. This emphasises the interest of looking at the A and B matrices more than at those on the diagonal, as proclaimed since the Introduction section.

An aspect of these sub-networks, or communities should be strongly emphasized. Although the number of triangles involving members of the different communities is very large and approximately equivalent, see above, it is found at this stage that there is apparently no homogeneous triangle involving the leaders of opinion if they belong to the DED community. We attribute this to a stronger rivalry in the DED community than in the IDP ideology. This is **hardly seen** if the analyses had been stopping at a tag-free set of agents, thus is an interesting argument in favor of examining networks with tagged nodes and biased links in further works.

In this respect the study of the local clustering coefficients indicates a low value for the inter-community networks. The above results indicate some leading opinion leaders or rather controversy makers. The average overlap index (AOI) [38] allows to extract from the clusters those persons which inside their community and with respect to the other are the centers of more attention, i.e. see Table 8.

Comments and suggestions on modelization of such society structure can be now in order. One may first consider the practical aspects resulting from the node characteristics, next those from links. The main opinion models in the literature consider the node state as somewhat independent of the link state,

merely carrying some information flow. The basic question is whether some stable phase exists and whether some phase transition can take place and if so under what circumstances, geometric, or "energetic" condition. from a

Since the networks are not scale free, nor random, since nevertheless there is some attachment effect to the community the spreading epidemics or its confinement should occur through a change in opinion, not of the hub (inflexible) nodes or agents, but of the agents at the border of one of the communities. This does not contradict Galam [39] assertion that to focus on convincing open mind agents is useless when there are so claimed incomplete or dubious data. Galam observes that *when the scientific evidence is not as strong as claimed, the inflexibles* (in whatever community) *rather than the data are found to drive the collective opinion of the population.* Whence a strong emphasis on node roles.

Practically this suggests one way to soften the controversy. *To produce inflexibles in one's own side is thus critical to win a public argument whatever the rigor cost and the associated epistemological paradoxes* [39].

Thus conducting a thorough analysis of (both) issues and to adopt a fair discourse are lose-out strategy ingredients. Adopting a cautious balanced attitude based on whatever scientific or not facts in contrast with an attitude of overstating arguments and asserting wrong statements, - which cannot be scientifically or *a priori* disregarded, is found to be necessary to eventually win a public debate. This is of interest in a democratic like system or meeting. However we conjecture that in a system in which the "votes" are taken over a long time scale, when the hubs are the inflexibles, as here, while the other agents have loose bounds, small connexions, and belong to heterogeneous triplets the change of opinion might occur. It is not obvious that *to adopt a cynical behavior is the obliged path to win a public debate against unfair and rigid opponents.* Indeed, how long would remain an opponent convinced of the rightness of the other attitude when he/she perceives the lies.

In view of the analysis and values reported in Table 1, it appears that there are 104 intercommunity links out of 526, whence $\simeq 0.20$ of the total amount, i.e. a proportion above the critical value 0.15 found in [12].

However, qualitatively, one can expect that the dynamics is driven by the collaboration and the intercommunity links, and not by a degree of freedom, like a "spin", attached to a node. Thus a model taking into account the number of interconnexions [3,12] would seem to be more appropriate. Our analysis indicates that there are 20% intercommunity link, -s much above the critical value 0.15 for which a transition to an ordered consensus state is possible according to the model in [3,12].

It is intuitively obvious that there is no "phase transition" to be expected in

the present system. Indeed the communities are pretty much behaving as in a very deep *potential well*. Their opinion can hardly be modified even through increasing the number of links or the intensity of the information exchange. These might even be counterproductive [39]. In other words, there is not much change/fluctuations in the node state, whatever the link weight or node degree. It can surely be said that a phase state change is likely more "easy" in economy and politics than in so called religious matter, - assuming a logical or scientific approach.

Therefore a third type of model comes in mind: the importance of the links in defining the node state and community makes us think of an analogy with ferro-electric models in Statistical Mechanics. Remember that the links carry a sense in such models, and the site weight depends on the sense of the arrows on the links out- or in-coming on the site. In so doing the interaction energy between two nodes depends on the configuration of these links at both connected nodes. It is thereby suggested to discuss where IDP-DED stability points defined through an order parameter connected to the set of triangles can be found in a phase diagram; the empirical observation of the emergence of homogeneous phases in large bi-community networks might help in at first a mean field approximation approach.

In more sophisticated models one could take into account that an opinion is not simply +1 and -1, as in so called "categorical" ones, but rather consider that the tag can take discrete values, i.e., with several "levels" as also suggested in [58], i.e. being a so called "ordinal" or continuous-like variable attached at a node [59].

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References

[1] D. Stauffer, Physica A **336** (2004) 1-5.

[2] R. Pastor-Satorras and A. Vespignani, *Evolution and Structure of the Internet : A Statistical Physics Approach*, (Cambridge University Press, Cambridge, 2004); R. Pastor-Satorras, M. Rubi, and A. Diaz-Guilera, *Statistical Mechanics of Complex Networks*, Lect. Notes Phys. **625** (Springer, Berlin, 2003)

[3] R. Lambiotte and M. Ausloos, J. Stat. Mech.: Theor. Exp. (2007) P08026

[4] *Empirical sciences of financial fluctuations. The advent of econophysics*, Tokyo, Japan, Nov. 15-17, 2000 Proceedings H. Takayasu, Ed. (Springer Verlag, Berlin, 2002) ; *Practical Fruits of Econophysics*, H. Takayasu, Ed., (Springer, Tokyo, 2006).

[5] *Econophysics and Sociophysics*, B.K. Chakrabarti, A. Chakraborti and A. Chatterjee, Eds. (Wiley-VCH, Weinheim, 2006).

[6] L.F. Costa, O.N. Oliveira, G. Travieso, A. Rodrigues, P.R. Villas Boas, L. Antiqueira, M.P. Viana, and L.E. Correa da Rocha, *Analyzing and Modeling Real-World Phenomena with Complex Networks: A Survey of Applications*, arXiv:0711.3199v2

[7] A. Pękalski, Physica A **252** (1998) 325-335.

[8] M. Girvan and M.E.J. Newman, *Proc. Natl. Acad. Sci. USA* **99** (2002) 7821-7826.

[9] M. Marsili, F. Vega-Redondo, and F. Slanina, *Proc. Natl. Acad. Sci. USA* **101** (2004) 1439-1442.

[10] S. Milgram, Psychol. Today **1** (1967) 60-67.

[11] P. D. Killworth, E.C. Johnsen, H. R. Bernard, G.A. Shelley, and C. McCarty, Social Networks **23** (1990) 289-312.

[12] R. Lambiotte, M. Ausloos, and J.A. Holyst, Phys. Rev. E **75** (2007) 030101(R).

[13] R. Lambiotte and P. Panzarasa, J. Informetrics **3** (2009) 180-190.

[14] T. Evans and R. Lambiotte, Phys. Rev. E **80** (2009) 016105.

[15] P. Fronczak, A. Fronczak, and J. A. Holyst, Phase transitions in social networks, $http://arxiv.org/abs/physics/$0701182

[16] I. Derenyi, I. Farkas, G. Palla, and T. Vicsek, *Phys. Rev. E* **69** (2004) 046117.

[17] D.J. Watts and S.H. Strogatz, Nature **393** (1998) 440-442.

[18] P. Svenson and D.A. Johnston, *Phys. Rev. E* **65** (2002) 036105.

[19] M. Ausloos and F. Petroni, Europhys. Lett. **77** (2007) 38002.

[20] Z. I. Dimitrova and N. K. Vitanov, *Phys. Lett. A* **272** (2000) 368-380.

[21] C. Buckley and J. Steele, *World Archaeology* **34** (2002) 26-46.

[22] S. Moss, *Proc. Natl. Acad. Sci. USA* **99** (2002) 7267-7274

[23] M.A. Porter, P.J. Mucha, M.E.J. Newman, and A.J. Friend, Physica A **386** (2007) 414-438.

[24] M.A. Porter, J.P. Onnela, and P.J. Mucha, Notices of the American Mathematical Society **56** (2009) 1082-1097 & 1164-1166.

[25] Xia Meng Si, Yun Liu and Zhen Jiang Zhang, Intern. J. Mod. Phys. C **20** (2009) 2013-2026.

[26] Ph. Ball, *Critical Mass: How One Thing Leads to Another*, Heinemann/Farrar Straus & Giroux (2004).

[27] This subject of intense interest is considered and conveniently approached in $http://www.talkorigins.org/origins/faqs-creationists.html$

[28] A. Garcia Cantù Ross and M. Ausloos, *Scientometrics* **80** (2009) 457-472.

[29] $http://www.arn.org/blogs/index.php/3/2007/10/14/science\_and\_intelligent\_design\_to\_be\_discussed$

[30] $http://www.brownalumnimagazine.com/november/december-2005/the-evolution-of-ken-miller.html$

[31] G. Bosetti, *La mia fede*, Reset, Marsilio Ed. (2008).

[32] W. W. Zachary, J. Anthropol. Res. **33** (1977) 452-473.

[33] H.R. Bernard, P.D. Kilworth, M.J. Evans, C. McCarty, and G.A. Shelley, Ethnology **27** (1988) 155-179.

[34] M.E.J. Newman, *Proc. Natl. Acad. Sci. USA* **98**, 404-409 (2001).

[35] I. Hellsten, R. Lambiotte, A. Scharnhorst, and M. Ausloos, Scientometrics **72** (2007) 469-486.

[36] M.E.J. Newman, Phys. Rev. Lett. **89** (2002) 208701.

[37] M.E.J. Newman, Phys. Rev. E **67** (2003) 026126.

[38] M. Gligor and M. Ausloos, Eur. Phys. J. B **63** (2008) 533-539

[39] S. Galam, *Public debates driven by incomplete scientific data: the cases of evolution theory, global warming and H1N1 pandemic influenza*, private communication

[40] R.T. Pennock, Ann. Rev. Genomics Hum. Genet. **4** (2003) 143-163.

[41] L.A.N. Amaral, A. Scala, M. Barthelemy, and H.E. Stanley *Proc. Natl. Acad. Sci. USA* **97** (2000) 11149-11152.

[42] C.M. Jarque and A.K. Bera, Intern. Stat. Rev. **55** (1987) 1-10.

[43] M.E.J. Newman, *The structure and function of complex networks*, arXiv:cond-mat/0303516 v1

[44] S. Redner, Eur. Phys. J. B **4** (1998) 131-134.

[45] F. Radicchi, S. Fortunato, and C. Castellano, Proc. Natl. Acad. Sci. USA **105** (2008) 17268-17272.

[46] J.P. Nolan, *Maximum likelihood estimation and diagnostics for stable distributions*, in O. Barndorff-Nielsen, T. Mikosh, and S. Resnick, editors, *Lévy Processes: Theory and application* (Birkhäuser, Boston, 2001).

[47] A.C.C. Shalizi and M. Newman, Power-law distributions in empirical data, E-print, arXiv:0706.1062 (2007).

[48] H.L. Seal, J. Institute of Actuaries **78** (1952) 115–121 .

[49] E.F. Fama, J. Business **38** (1965) 34-105.

[50] G Rotundo, M Navarra, Physica A **382** (2007) 235-246.

[51] V. Batagelj and A. Mrvar, *Pajek, Program for Analysis and Visualization of Large Networks Reference Manual*, Fig. 15, p. 52

[52] R. Milo, S. Shen-Orr, S. Itzkovitz, N. Kashtan, D. Chklovskii, and U. Alon, Science **298** (2002) 824-827.

[53] F. O. Redelico, A.N. Proto, and M. Ausloos, Physica A **388** (2009) 3527-3535

[54] I. Hellsten, R. Lambiotte, A. Scharnhorst, and M. Ausloos, *Scientometrics* **72** (2007) 469-486.

[55] G. Tibely and J. Kertesz, Physica A **387** (2008) 4982-4984.

[56] F. Radicchi, C. Castellano, F. Cecconi, V. Loreto, and D. Parisi, *Proc. Natl. Acad. Sci. USA* **101** (2004) 2658-2663.

[57] H. R. Bernard, P. D. Killworth, C. McCarty, G.A. Shelley, and S.Robinson. Social Networks **12** (1990) 179-215.

[58] M. Ausloos, *On religion and language evolutions seen through mathematical and agent based models*, Proceedings CHESS 2009 conference, in press.

[59] see, e.g., $http://www.oswego.edu/\ sims\ rp/stats/variable\_types.htm$; or $http://www.ats.ucla.edu/stat/mult\_pkg/whatstat/nominal\_ordinal\_interval.htm$

[60] see $http://www.discovery.org/csc/$

[61] see $http://www.ncseweb.org/$

Appendix A: Historical perspective

Let us briefly cite [28] to put some historical perspective: *One may distinguish two main opinion groups about the subject of the origin of the universe and life. The first group holds the scientific consensus and in particular Darwin evolution theory as a valid basis, whilst the second is formed by people adopting a theistic (in the present context, biblical) view where natural processes are conceived as occurring out of the purposeful will of some supra natural entity. Amid them some belong to a historical movement, so called creationism, which aims to refute and overturn Darwin theory. Such organized opposition to evolution has found most of its adherents in different Christian traditions, actively engaged in promoting their values at the core of society. The influence of some of these religious groups has been especially relevant in some regions of USA and is introduced in other parts of the world, for reasons not discussed at this level.* and *Specifically, the ID proponents* (IDP) *have employed concepts of information theory, thermodynamics and molecular biology in seeking for evidences of an intelligent blueprint underlying the complexity observed in biological systems. Yet, none of the IDP arguments has been validated by most of the scientific community. In spite of this fact, the ID movement has further developed: since the second half part of the 1990s, in good measure thanks to the support and headquarter brought by the Discovery Institutes Center for Science and Culture (CSC)*[60].

The increasing activity and impact of the ID movement has impelled the reaction of social and scientific organizations around the world. Among the most important ones, the non-profit organization National Center for Science Education (NCSE) [61] plays a relevant role in coordinating the activity of people defending the teaching of evolutionary biology in the USA. Hereafter we refer to the international group of people fighting ID as Darwin evolution defenders (DED).

Appendix B: IDP-DED matrix

In this Appendix we give the adjacency matrices of interest: C, A, B, D.

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